The birthday paradox

In a room of just **23 people**, the odds that two of them share a birthday are better than even. Most people guess you would need closer to 180. The gap between that guess and the truth is what makes this a paradox, and it is a clean little demonstration of why exponential intuition fails us.

The math

It is easier to count the chance that *no one* shares a birthday and subtract from one. Lining people up one by one, each must avoid all previous birthdays, giving the chance of a match P:

$$P = 1 - \frac{365!}{365^n * (365 - n)!}$$

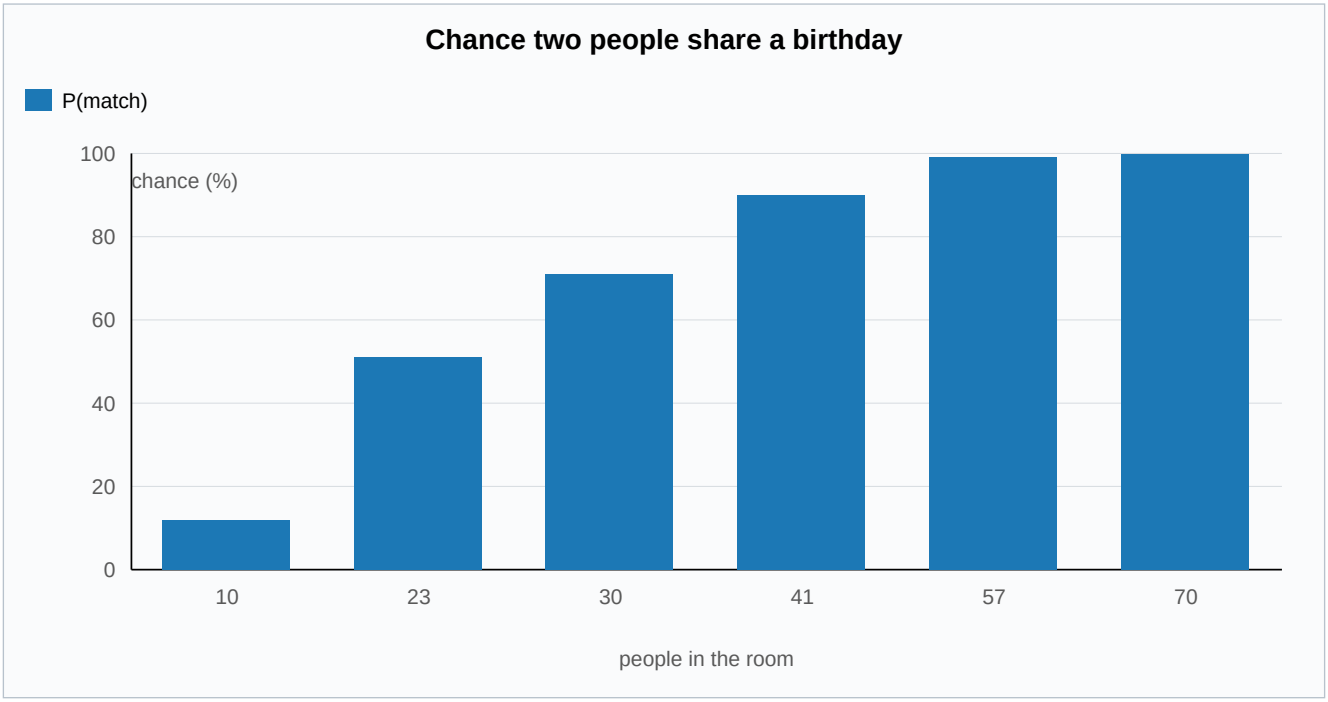
The reason it climbs so fast is that the number of *pairs* who could match grows with the square of the group:

$$\frac{n(n - 1)}{2}$$

At 23 people that is already 253 pairs, each a separate chance to collide.

The numbers

People	Pairs	Chance of a shared birthday
10	45	12%
23	253	51%
30	435	71%
41	820	90%
57	1596	99%
70	2415	99.9%



Past about 60 people a shared birthday is all but guaranteed, yet you would need 367 people to *guarantee* it by the pigeonhole principle. The curve gets to near-certainty long before it gets to certainty.

Every formula and the chart above are static SVG produced at build time: no MathJax, no chart library, no JavaScript.